

**Semester One Examination, 2020**

**Question/Answer Booklet**

**MATHEMATICS**

**SOLUTIONS**

**METHODS**

**ATAR Year 12**

**Section One:**

**Calculator-free**

Student Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Please circle your teacher’s name

**Teacher: Miss Long Miss Rowden Ms Stone**

**Time allowed for this paper**

Reading time before commencing work: 5 minutes

Working time for paper: 50 minutes

**Materials required/recommended for this paper**

***To be provided by the supervisor***

Number of additional

answer booklets used

(if applicable):

This Question/Answer Booklet

Formula Sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Suggested working time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  |  |  |  | **Total** | 100 |

**Instructions to candidates**

1. The rules for the conduct of the ATAR course examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Supplementary pages for the use planning/continuing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free 35% (52 Marks)**

This section has eight (8) questions. Answer **all** questions. Write your answers in the spaces

provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 (5 marks)

Determine the area bounded by the line and the parabola .

|  |
| --- |
| **Solution** |
| Intersect when  Bounded area |
| **Specific behaviours** |
| ü equates functions and simplifies   bounds of integral   writes definite integral   antidifferentiates   correct area |

Question 2 (7 marks)

Determine the following

(a) 

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü correct integration of trig term   correct integration of term involving e  - if no + *c* (penalise once only – either a or b or for both) |

(b) 

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü correct numerator   correct denominator |

(c) Evaluate 

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü correct antiderivative   evaluates constant and writes equation |

Question 3 (8 marks)

Determine

(a) when .

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü indicates correct use of chain rule   correct derivative (any form) |

(b) when .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  or correct   correct derivative in terms of   correct value |

(c) when .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü indicates correct use of quotient rule   correct derivative   correct value, simplified |

Question 4 (7 marks)

(a) Find *x* if:

(i) **

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü rewrites logarithmic equation as exponential with prime bases  ü solves correctly for *x* |

(2 marks)

(ii) **

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü rearranges to make *x* the subject  ü applies the logarithm laws to simplify to a single logarithm  ü solves correctly for *x* |

(b) Simplify **

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü expands each term  ü final answer |

(2 marks)

Question 5 (5 marks)

The graph of has a stationary point at and , where is a constant.

Determine the interval over which and .

|  |
| --- |
| **Solution** |
| Concave up:  Other stationary point:  Hence when .  Required interval: . |
| **Specific behaviours** |
| ü value of   interval where   second stationary point   interval where   correct interval |

Question 6 (5 marks)

A curve, defined for , passes through the point and its gradient is given by

(a) Verify that is a stationary point, determine the value of the second derivative at and hence describe the nature of the stationary point. (3 marks)

|  |
| --- |
| **Solution** |
| , so is a stationary point.  , so is a local minimum. |
| **Specific behaviours** |
| ü simplifies to three integers that sum to zero   correct value of second derivative   states correct nature |

(b) Determine the equation of the curve. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü correct antiderivative   evaluates constant and writes equation |

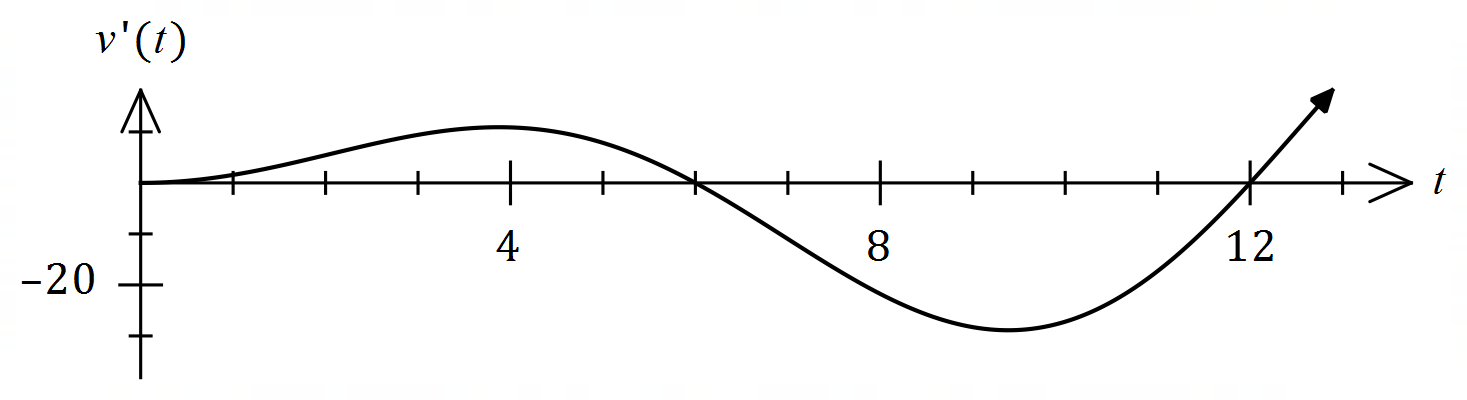
Question 7 (7 marks)

(a) Determine an expression for .

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü correct use of product rule   correct derivative |

The volume of water in a tank, litres, is changing at a rate given by , where is the time in hours. The rate of change is shown in the graph below.



(b) Using the result from part (a) or otherwise, determine the change in volume of water in the tank between and hours.

|  |
| --- |
| **Solution** |
| 1. Using (a):  2. And so:  3. Hence: |
| **Specific behaviours** |
| ü indicates required definite integral   line 1 - uses part (a)   line 2 - expression to evaluate integral   line 3 - antidifferentiates ready for substitution   correct change in volume, with units |

(5 marks)

Question 8 (8 marks)

Initially, particle is stationary and at the origin. Particle moves in a straight line so that at time seconds its acceleration cms-2 is given by where .

(a) Determine the speed of after second.

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  indicates is integral of  ü expression for velocity   correct speed |

(b) Determine the speed of when it returns to the origin.

(5 marks)

|  |
| --- |
| **Solution** |
| Require change in displacement for  Hence speed is cm/s. |
| **Specific behaviours** |
|  obtains expression for in terms of   equates   solves for   obtains velocity   correct speed, with units |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_